Random porosity in granular media

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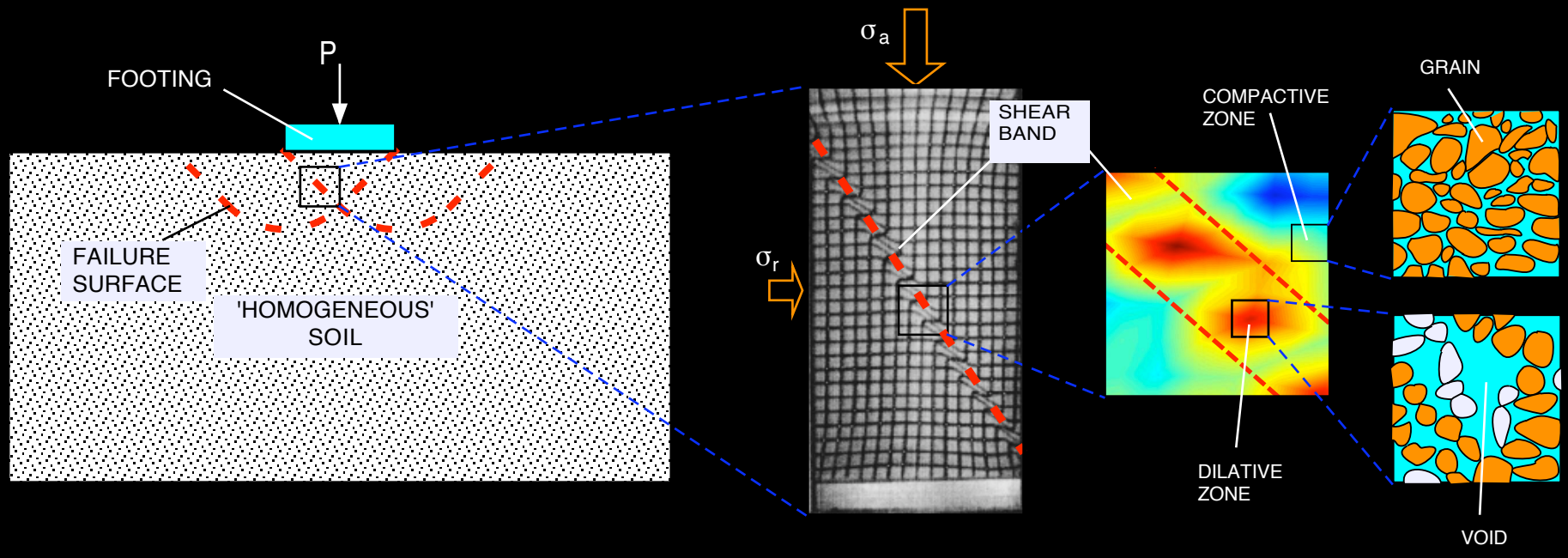
Sands across scales



FIELD SCALE | SPECIMEN SCALE | MESO SCALE | GRAIN SCALE

LOG (m) >1 0 -1 -2 -3

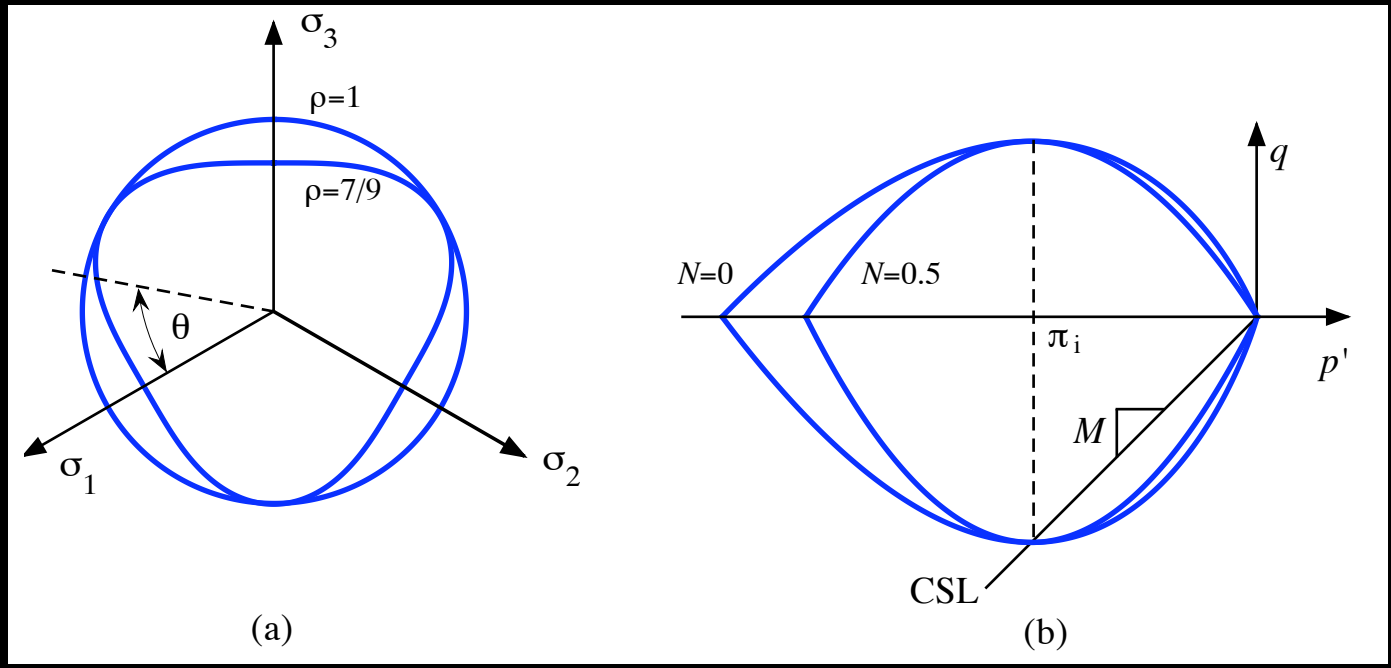
Objectives

- Couple state-of-the art plasticity models for granular media with geostatistical tools
- Account for inherent heterogeneities at the meso-scale and study their impact on stability of granular media
- Use CT data and experiments to calibrate/ train and validate models

Constitutive framework

- Elastic regime based on hyperelasticity
- Plastic response based on CSSM
- Relative density governs hardening/softening and dilation (very important in sands)
- Model captures most salient features of sand behavior

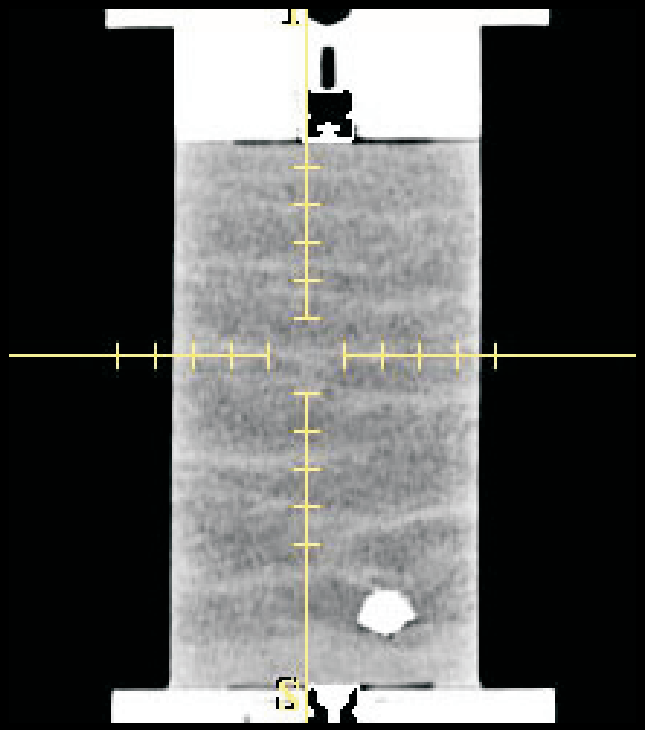
Plasticity formulation



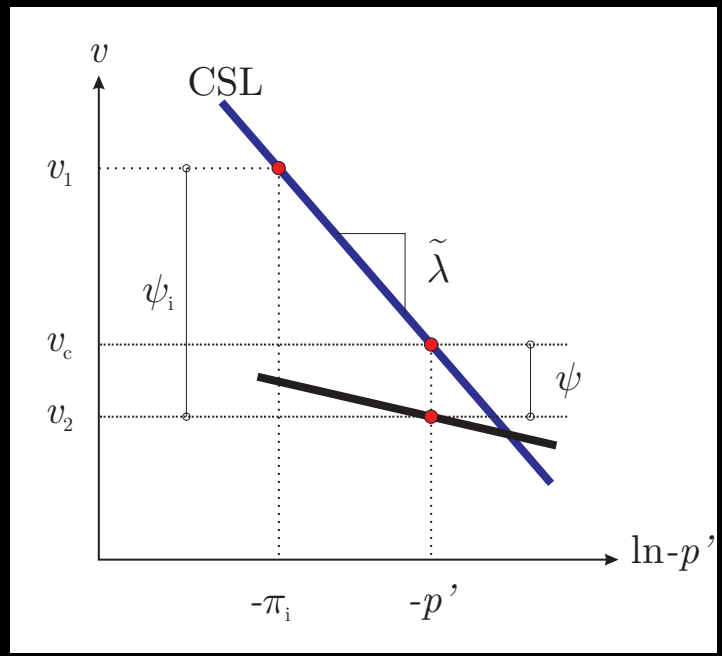
$$\dot{\boldsymbol{\sigma}} = \boldsymbol{c}^{\mathrm{ep}} : \dot{\boldsymbol{\epsilon}} \qquad \dot{\boldsymbol{\epsilon}} = \dot{\boldsymbol{\epsilon}}^{\mathrm{e}} + \dot{\boldsymbol{\epsilon}}^{\mathrm{p}} \qquad \dot{\boldsymbol{\epsilon}}^{\mathrm{p}} = \dot{\lambda} \frac{\partial Q}{\partial \boldsymbol{\sigma}}$$

Imaging and modeling

ct scan

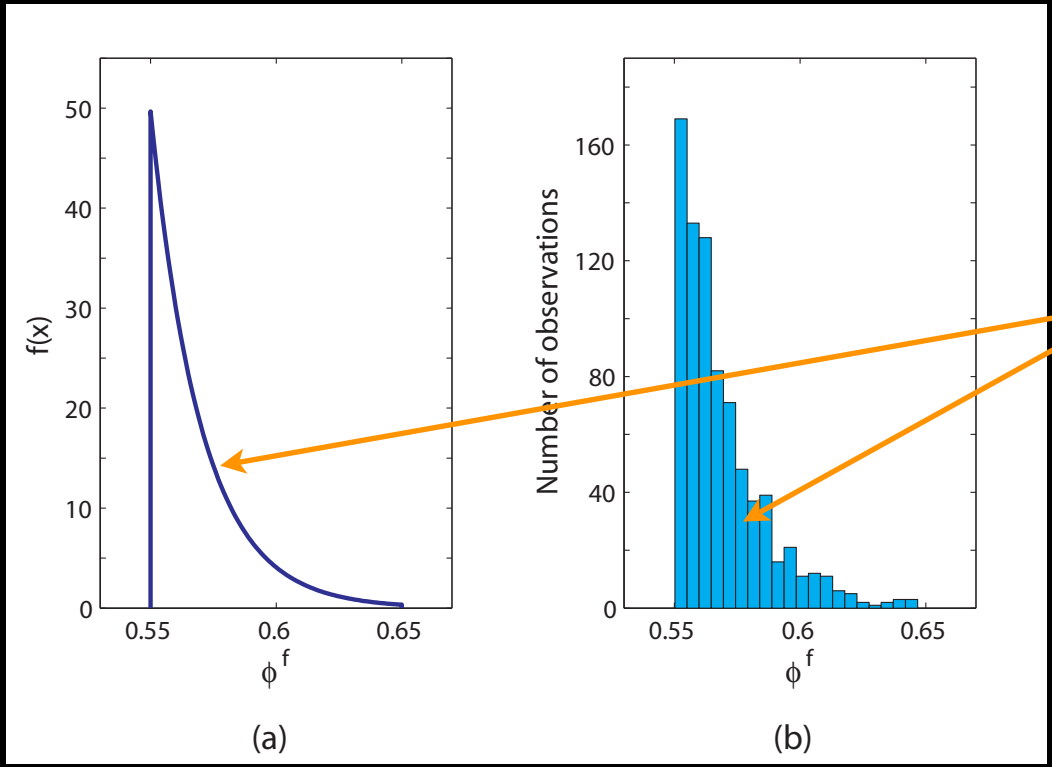


ψ_i



void ratio treated as only random field

Void ratio: exponential



truncated exponential

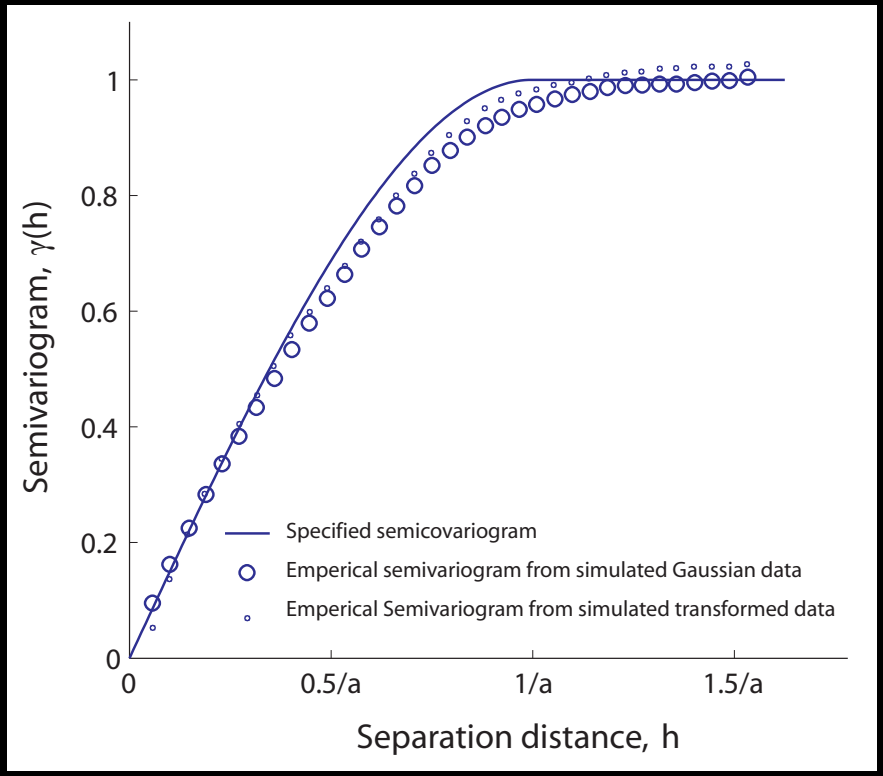
$$f(x) = \frac{P(x \leq \phi^{\mathrm{f}} < x + dx)}{dx} = \begin{cases} 49.66e^{-50(x-0.55)} & \text{if} \quad 0.55 \leq x < 0.65 \\ 0 & \text{otherwise} \end{cases}$$

Sequential simulation

- Sequential conditional simulations
- Flexible: allows for observed data inclusion
- Starts at random point, conditional simulations at other random points
- Simulate $z \sim N$ and transform

$$\left[\frac{Z^{(i)}}{\mathbf{Z}^{(sampled)}}\right] \sim N\left(\left[\frac{0}{\mathbf{0}}\right], \left[\begin{array}{c|c} 1 & \boldsymbol{\Sigma}_{12} \\ \hline \boldsymbol{\Sigma}_{21} & \boldsymbol{\Sigma}_{22} \end{array}\right]\right) \qquad x = F\left(\Phi^{-1}(z)\right)$$

Semivariogram



$a/b = 1$

isotropic case

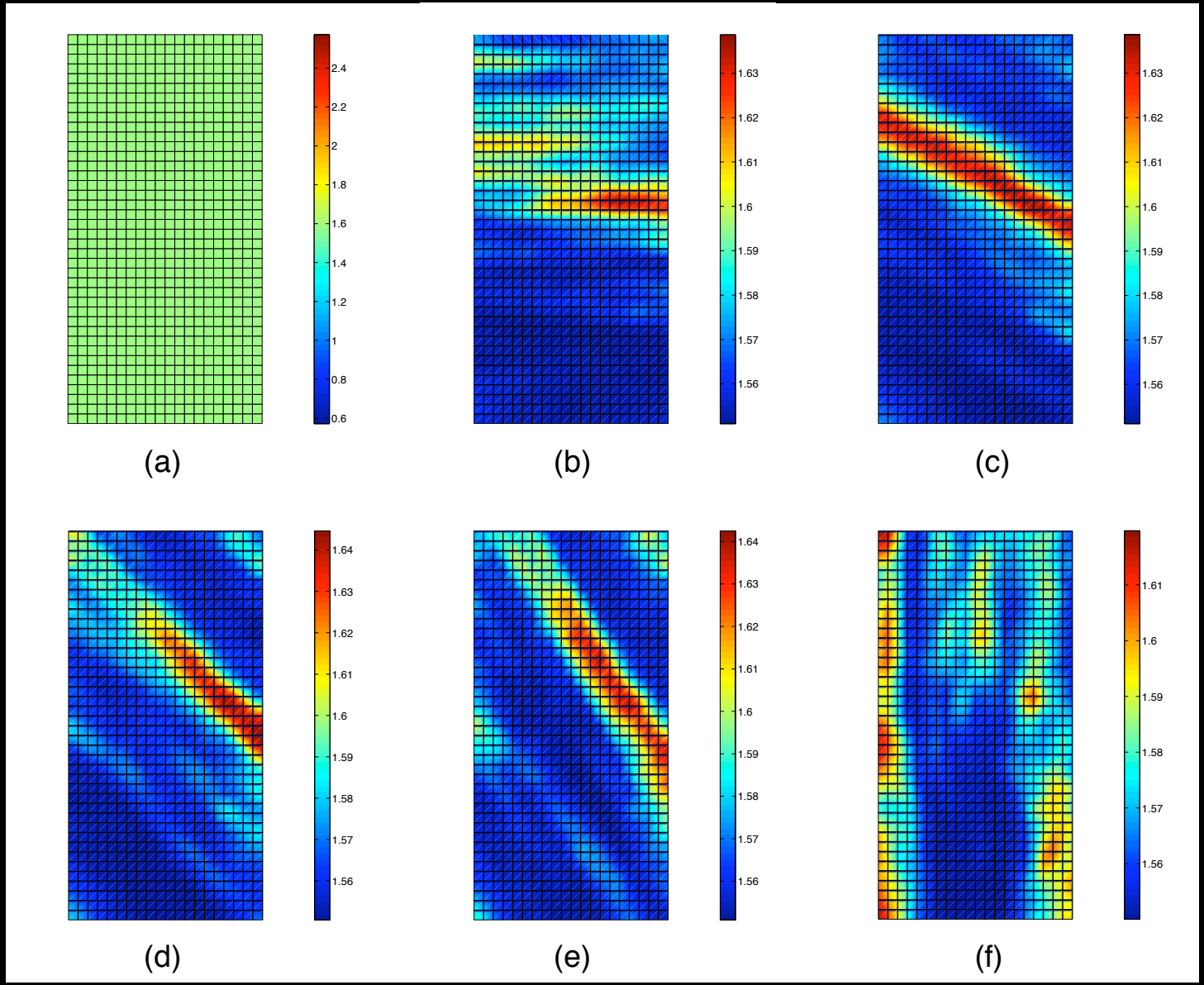
$$\gamma(\mathbf{h}) = \begin{cases} \frac{3}{2}\left(\left(\frac{h_1}{a}\right)^2 + \left(\frac{h_2}{b}\right)^2\right)^{\frac{1}{2}} - \frac{1}{2}\left(\left(\frac{h_1}{a}\right)^2 + \left(\frac{h_2}{b}\right)^2\right)^{\frac{3}{2}} & \text{if} \quad \left(\frac{h_1}{a}\right)^2 + \left(\frac{h_2}{b}\right)^2 \leq 1 \\ 1 & \text{if} \quad \left(\frac{h_1}{a}\right)^2 + \left(\frac{h_2}{b}\right)^2 > 1 \end{cases}$$

Numerical simulations

a/b	orientation	Number of simulations		
		20×40	20×40–normalized	10×20
1	n.a.	12	1	1
10	0	12	1	-
10	30	12	1	-
10	45	12	1	-
10	60	12	1	-
10	90	12	1	-
100	0	12	1	1
100	30	12	1	1
100	45	12	1	1
100	60	12	1	1
100	90	12	1	1

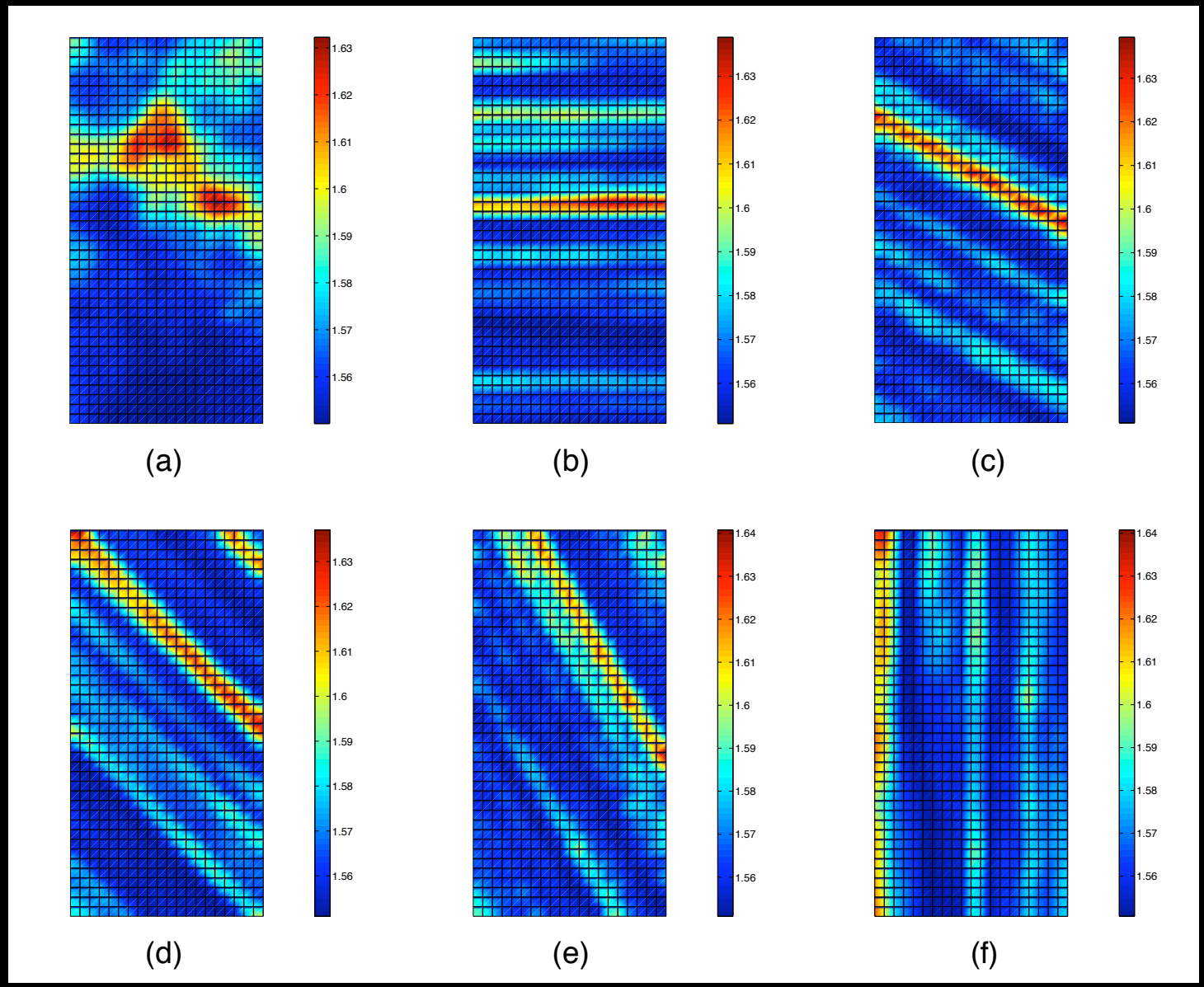
initial specific volume v_0

$a/b = 10$

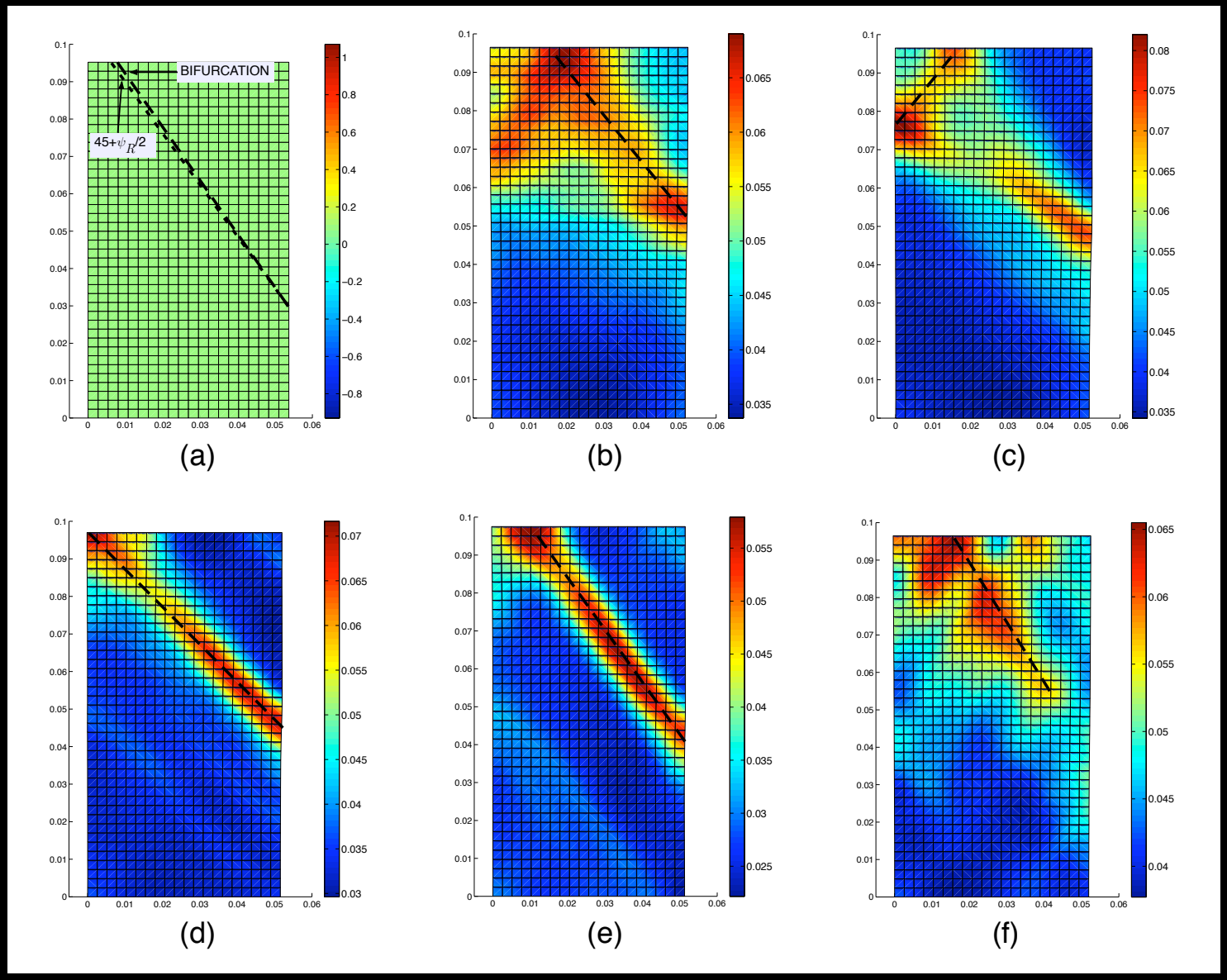


initial
specific
volume
v_0

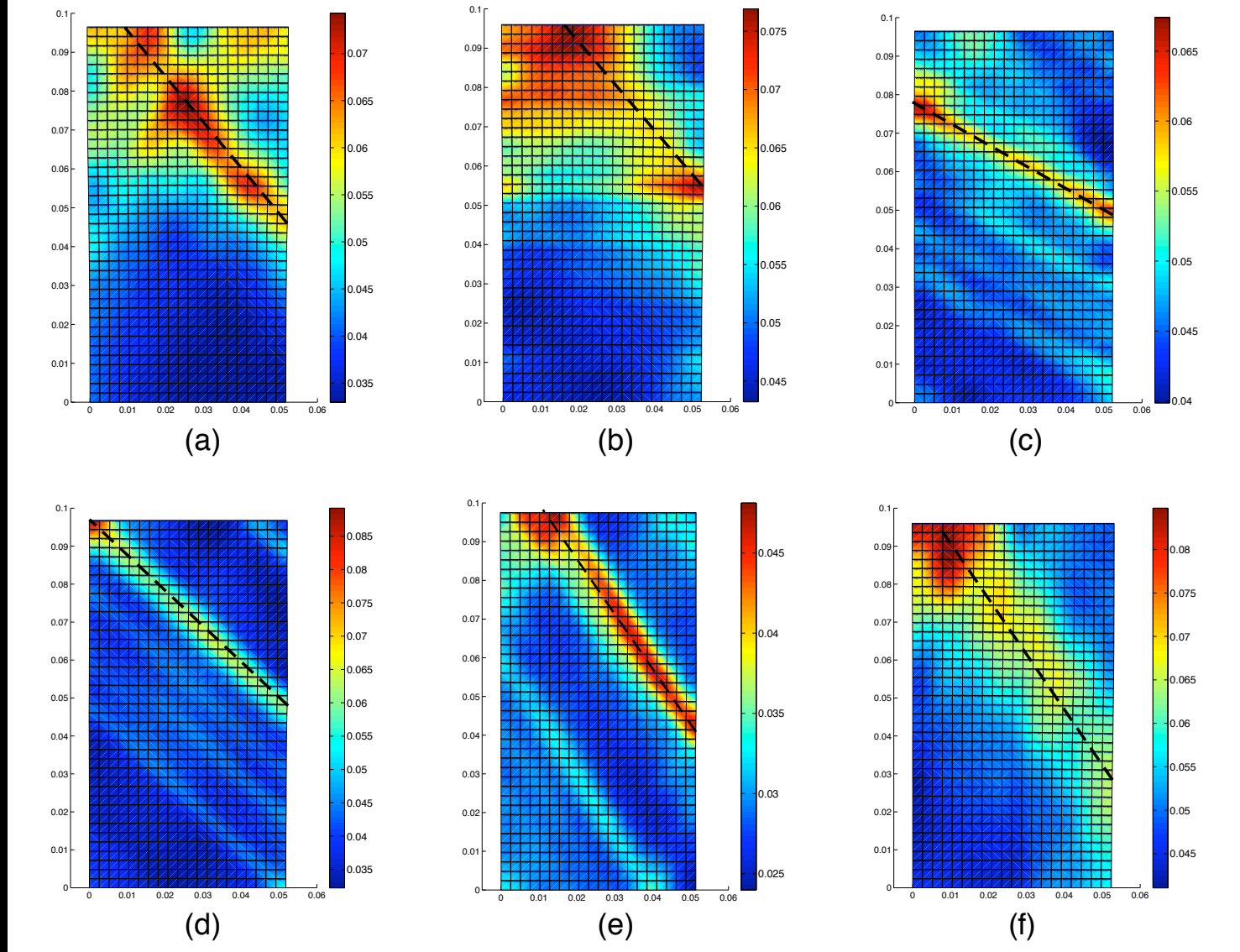
$a/b = 100$



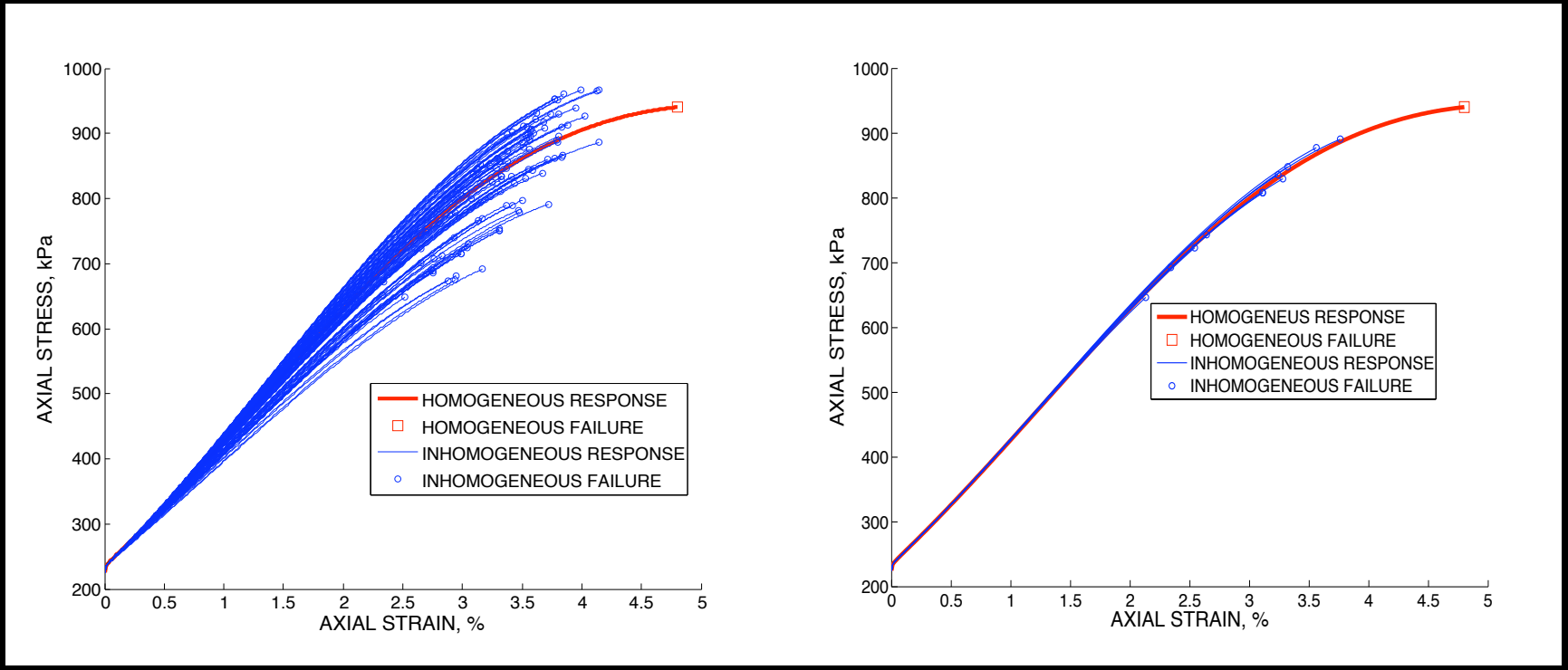
Bifurcation $a/b = 10$



Bifurcation $a/b = 100$



Load spread

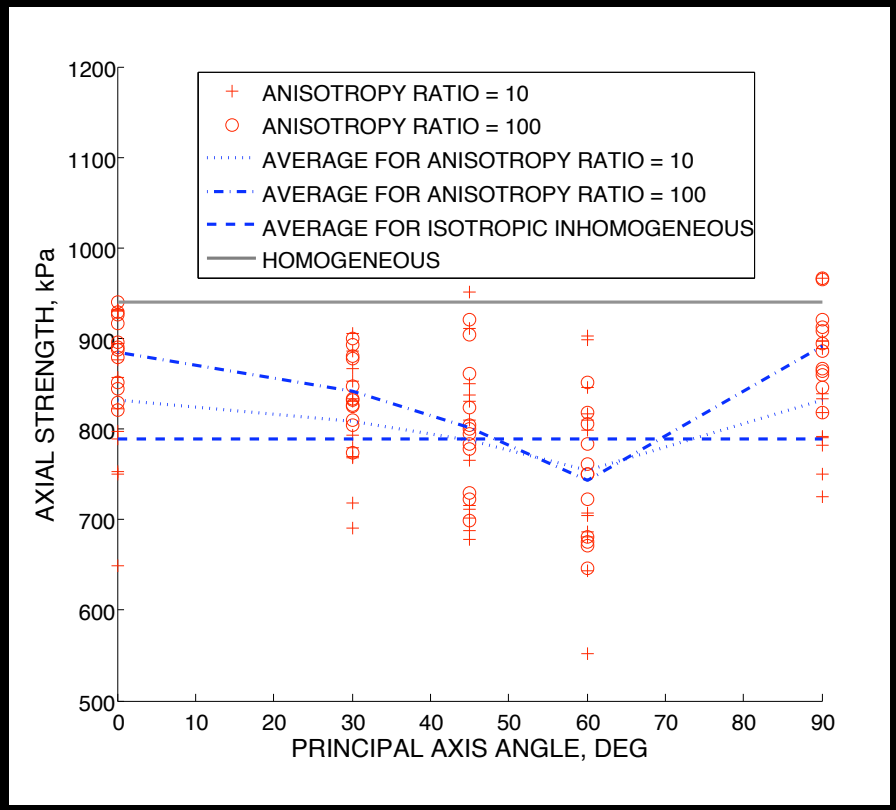


non-normalized

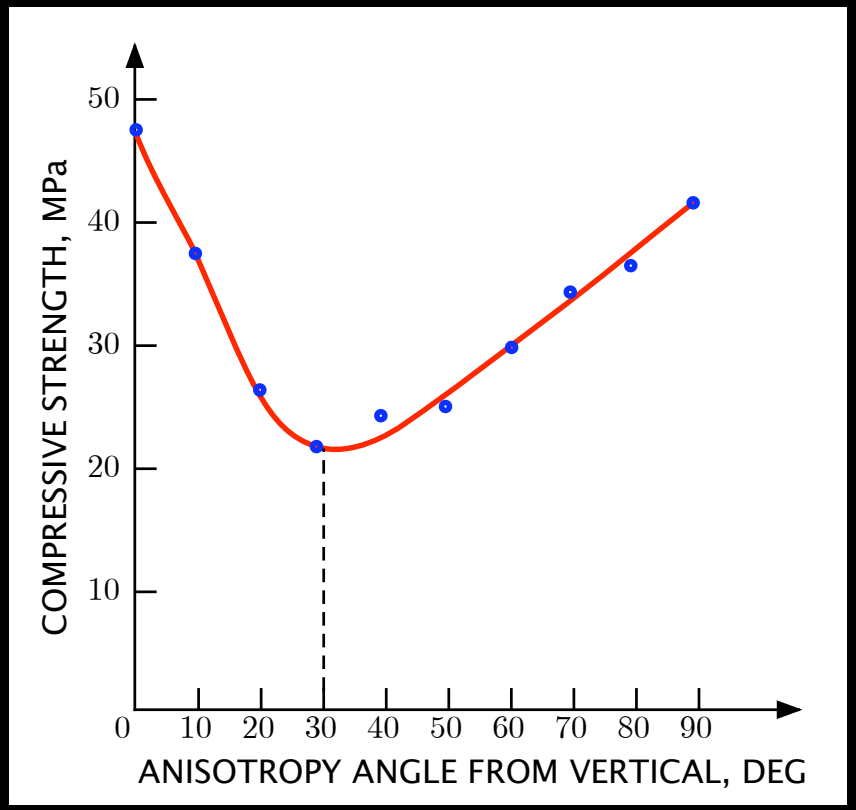
normalized

failure $\Leftarrow$ $\det A = 0$

Cf. anisotropic rocks

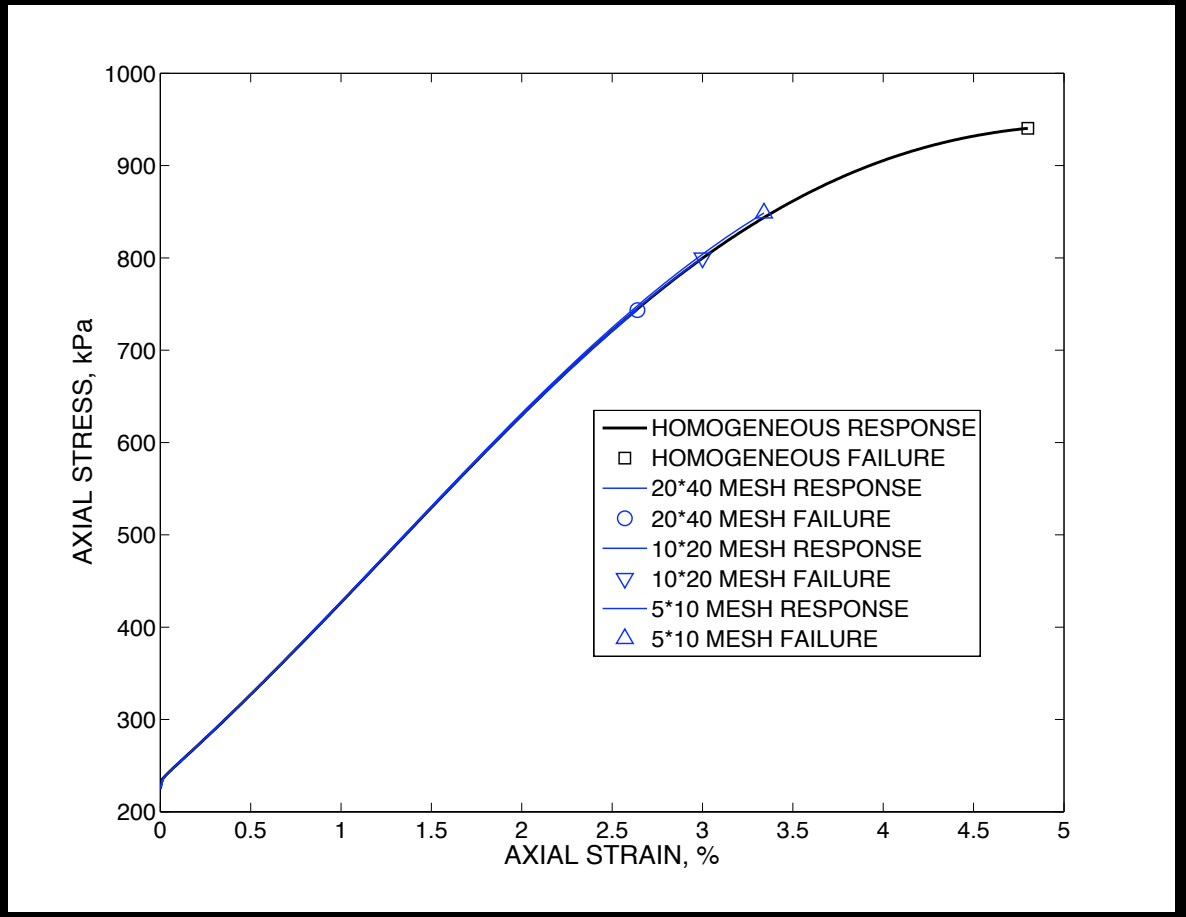


computations



anisotropic rocks

Mesh coarsening



Conclusions

- Elastoplasticity coupled with geostatistics
- CT data used to ‘inform’ model
- Isotropic and anisotropic porosity considered
- Heterogeneity accelerates instability
- Axial strength depends on degree and orientation of anisotropies

The future: multi-scale