LIQUEFACTION INSTABILITY

CAN WE PREDICT?
LIQUEFACTION CRITERION

REFERENCES: ANDRADE [2007], BORJA [2006]
HILL'S POSTULATE

\[ [\dot{\sigma}] : [\dot{\epsilon}] = 0 \]

**LOSS OF UNIQUENESS**

**JUMP IN RATE STRAIN FIELD**

\[ [\dot{\epsilon}] := \dot{\epsilon}^* - \dot{\epsilon} \]

**JUMP IN STRESS RATE FIELD**

\[ [\dot{\sigma}] := \dot{\sigma}^* - \dot{\sigma} \]

**POTENTIALLY**

**DUPLICATE SLNS**

\[ \dot{u}^* \text{ AND } \dot{u} \]
Saturated Conditions

Balance of Mass

\[ \nabla \cdot [\dot{u}] = -\nabla \cdot [q] \]

\[ [q] = 0 \]

Undrained Bifurcation

Incompressible Mass Conservation

\[ \nabla \cdot [\dot{u}] = \text{tr}[\dot{\epsilon}] = 0 \]
EFFECTIVE STRESS

\[
\dot{\sigma} = \dot{\sigma}' - \dot{p} \cdot 1
\]

EFFECTIVE STRESS
PORE PRESSURE

\[
\dot{\sigma} : \dot{\varepsilon} = \dot{\sigma}' : \dot{\varepsilon} - \dot{p} \cdot \text{tr} \dot{\varepsilon}
\]

\[
\dot{\sigma} : \dot{\varepsilon} = \dot{\sigma}' : \dot{\varepsilon}
\]
LIQUEFACTION CRITERION

\[
\dot{\sigma}' = c^{ep} : [\dot{\epsilon}]
\]

CONSTITUTIVE RELATION

LOSS OF POINT-WISE STABILITY

\[
\dot{\sigma}' : [\dot{\epsilon}] = [\dot{\epsilon}] : c^{ep} : [\dot{\epsilon}] = 0
\]

\[
H_{crit} = -K \frac{\partial F}{\partial p'} \frac{\partial Q}{\partial p'}
\]
PLASTICITY MODEL FOR SANDS

REFERENCE: ANDRADE & BORJA [2007]
\[ F = q + p' \eta \]

\[ p' = \frac{1}{3} \text{tr} \sigma' \]

\[ q = \sqrt{\frac{3}{2}} \| \text{dev} \sigma \| \]

\[ \eta = \eta(p', p_i, M, N) \]
PLASTIC FLOW & HARDENING

NONASSOCIATIVE FLOW

\[ \dot{\varepsilon}^p = \dot{\gamma} \frac{\partial Q}{\partial \sigma'} \quad Q = q + p'\bar{\eta} \]

\[ \bar{\eta} = \bar{\eta}(p', \bar{p}_i, M, \bar{N}) \]

\[ H = hM \left( \frac{p'}{p_i} \right)^{1/(1-N)} (p_i - p_i^*) \]

\[ H > 0 \quad \text{if} \quad p_i > p_i^* \quad \text{HARDENING MODULUS} \]

\[ H < 0 \quad \text{if} \quad p_i < p_i^* \quad \text{SOFTENING} \]
MODEL PREDICTIONS

CONSTANT POROSITY, DIFFERENT PRESSURE
DIFFERENT POROSITY, CONSTANT PRESSURE

REFERENCE: ANDRADE [2007]
$p'_0 = -100 \text{ kPa}$

Experiments from Doanh et al. [1997]
TXC AT CONSTANT $v_0$

$p_0' = -200 \text{ kPa}$

$p_0' = -300 \text{ kPa}$

EXPERIMENTS FROM DOANH ET AL. [1997]
THE INSTABILITY LINE

LADE & PRADEL [1990]

EXPERIMENTS FROM DOANH ET AL. [1997]
PORE PRESSURE BUILD-UP

DEVIATORIC STRESS, kPa vs AXIAL STRAIN

NORMALIZED PORE PRESSURE

INCREASING CONFINEMENT PRESSURE
EFFECT OF INITIAL DENSITY

LOOSEST SAMPLE

MEDIUM LOOSE SAMPLE

\[ D_R = 37.8\% \]

\[ D_R = 45.9\% \]

EXPERIMENTS FROM VAID & CHERN [1983]
NOT EVERYTHING LIQUEFIES

**Denser Sample** \( D_R = 48.2\% \)

---

Experiments from Vaid & Chern [1983]
LIQUEFACTION: BVPS

REFERENCE: ELLISON & ANDRADE [2008]
LIQUEFACTION IN 2D (QUASI-STATIC)
QUASI-STATIC LIQUEFACTION

- LIQUEFACTION CRITERION
- DEVIATORIC STRAINS
- PORE PRESSURES
QUASI-STATIC LIQUEFACTION

- LIQUEFACTION CRITERION
- DEVIATORIC STRAINS
- PORE Pressures
QUASI-STATIC LIQUEFACTION

- LIQUEFACTION CRITERION
- DEVIATORIC STRAINS
- PORE PRESSURES
THE SUBMERGED SLOPE FAILURE

WATER TABLE

APPLIED LOAD

5.0 m

10.0 m

10.0 m

18.7 m

17.3 m

20.0 m

30°
THE SUBMERGED SLOPE FAILURE

(a) Pore Pressure (in kPa)

(b) Deviatoric Strain

(c) $H-H_{\text{crit}}$

SLIP SURFACE
CONCLUSIONS

- Critical hardening modulus signals liquefaction
- Model accurately predicts so-called instability line
- Liquefaction is density-dependent
- Predictive = physics + accurate constitutive model
- Model can predict instabilities at the field scale
REFERENCES

ANDRADE [2007]. GÉOTECHNIQUE, IN REVIEW

ANDRADE & BORJA [2006]. IJNME, 67:1531-1564

BORJA [2006]. ACTA GEOTECHNICA, 1:211-224

DOANH ET AL. [1997]. MECH. OF COHES-FRICT MATLS, 2:47-70

ELLISON & ANDRADE [2008]. JGGE, IN REVIEW

VAID & CHERN [1983]. SOILS & FOUNDATIONS, 23:47-60